

Knowledge the Science

Result from experiment and appropriate laws impart knowledge called **Science**.

Physics is the science that impart knowledge needed for Engineering

Desires and needs of human being is fetched by the **Engineering knowledge**

Mechanics is a science that tells the behavior of bodies in motion and at rest due to influence of external force.

Mechanics

Fundamental Engineering Subject essential for **design of machinery**.

Mechanics is basic in **Physics and Astronomy**

Design of machinery is an important task in which structure of machines either moveable or stationary has to analyzed for its **failure and deformation**. This structural analysis is being done by various **techniques** such as **analytical method** and **experimental method** and **numerical method**.

The numerical analysis involves various solution includes **approximation method** and analytical and **finite element method** analysis. The finite element analysis has got various methods and techniques about which we are going to see in details here.

Engineering Analysis

Mobile Machineries

Usually some machinery are **operated at same place** they are called **Stationary Machineries**. Whereas some other machineries are working as **moving machines** they are called **mobile type machinery**.

Ex. Of Stationary machines : Mills and machine tools like lathes , planning machine, milling machines .

Ex. Of mobile machines : automobiles , trains, ships etc.

Structures

There are some fabricated items which are not having any movable parts and usually they represent structures.

Ex – Truss, beam, etc.

Engineering Analysis

To find deformation(change of dimensions), induced stresses on various parts of machineries and structures we adapt some techniques that are called Engineering Analysis. There are various techniques adopted and they are called methods of engineering analysis.

The techniques or suitable method employed for Engineering analysis depend on Size of machinery(small or big), Nature of its shape (simple or complicated).

Purpose of Engineering Analysis

Whatever type of machinery or structure it may be it should be operated with in its allowable loads carrying capacity or in other words it should function within its yield point or breaking point that is within its withstanding capacity.

If it is not loaded within its load carrying capacity the structure or machine parts will deform if it is further loaded it will fail function wise or break at a point material wise.

For avoiding such failure structures and machine parts should be loaded far less load than their maximum load carrying capacity or ultimate strength.

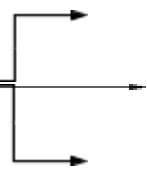
Hence we have to determine

1. Type and quantity of load
2. Location of loading
3. Developed stress
4. Permissible deflection
5. Vibration properties
6. Pressure and temperature variations
7. Other failure like crack formation and leanness etc.

Apart from all these especially for Human safety and Economical usage this analysis is carried out.

Methods of Engineering analysis

Engineering products are to be analyzed before its production. Methods involved are

1. Experimental methods
 2. Analytical method
 3. Numerical methods
- 
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graph LR; NM[3. Numerical methods] --> FAM[Functional Approximating method]; NM --> FEM[Finite Element method]; NM --> FDM[Finite Difference method];
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### **Finite Element method**

Mr. Clough therefore arrived a conclusion in his invention the finite element analysis particularly in structural elements of continuum it is divided into a finite number of elements having finite dimensions and reducing the continuum having infinite degree of freedom to finite degrees of freedom for analysis of structures

## Advantages and disadvantages of Finite Element Method

### ADVANTAGE

Mainly the physical problems intractable and complex for any closed bound can be analyzed by this method.

1. Efficiently applied to irregular geometry to get solution.
2. Any type of boundary objects can be analyzed
3. Anisotropy and Inhomogeneity of material can also be easily considered in this method.
4. Any type of loading can be considered and can be solved.

### DISADVANTAGE

1. There are many type of problems where some other method can solve efficient than the finite element method
2. The cost involved in this method is very high.
3. In many cases of Vibration and stability the cost of analysis by this method should be avoided .
4. The approximation used in development of element stiffness naturally in this method. No element will represent all possible behavior pattern equally So compensation in one element causes distribution of another behavior.
5. The stress values may vary by 25% from fine mesh analysis to average mesh analysis.
6. The aspect ratio may affect the final results.(longer to smaller dimension of elements)
7. Interpretation of output problem arises in this method. So much of axial force and moment etc. are to be analyzed to produce a stack of output in some cases in frame analysis. Even a smaller problem in this method would generate so many numbers that has to be considered. Graphical representation of displacements and stresses to specific application require expertise.

In a Nut shell

There are four principles areas directly connected with the finite element method

1. Derivation of the Theory
2. Idealizing actual problem to approximate finite element problem
3. Computer program development for applying finite element theory
4. Investigation of data information processing and numerical methods needed to compute finite element solution

The major assumption on which the development is based should be understood and known to the engineers applying finite element method. Significance of approximation assumed and built in the method and the limitations of the analytical models developed should be known. This knowledge in finite element method reveals the difference between the good and bad disaster.”

### **Three type of forces acting on structure**

**Body Force** – distributed force acting on every elemental volume of the body. Force / Unit volume

**Traction Force** - Distributed force acting on the surface of the body – Force / Unit area

**Point Load** – Force acting at a particular point to cause displacement  
Force / point

## **FINITE ELEMENT METHODS PRINCIPLES...**

**Finite Element Method** is numerical method employed for obtaining solutions to many problems

Encountered in Engineering and Mathematical Physics field.

## **Engineering fields**

- 1 Structural analysis
- 2 heat transfer
- 3 Fluid flows
- 4 Mass transfers
- 5 Electromagnetic potential

## **Application of this FEM analysis in**

1. Design of Automobiles
2. Air frames
3. High rise building
4. Space crafts
5. Heat engines
6. Electric motors
7. Bearing ....etc.

## **What is an Analytical method?**

Analytical solution is a **mathematical expression** that gives the values of desired unknown quantity at any location in a body. (Body i.e. Whole structure or any physical (area) system of interest. )

Consequently it is valid for infinite number of locations in that body.

**Analytical method** is employed for solving simple idealized problems.

## **Due to**

- 1 Complicated Geometry
- 2 Complicated Loadings
- 3 Complicated Material properties

It is very difficult or not at all possible to get solutions or cent percent correct solutions are not possible some times in engineering problems practically by analytical methods.

To overcome this method or alternative technique called NUMERICAL Methods used to find

At least Approximate Solutions. But it is accepted solution for complicated problems arise in engineering structural analysis.

### **Some numerical methods are**

1. Variational method or Rayleigh –Ritz method
2. Weighted residual methods
3. Finite element method
4. Finite difference method
5. Finite volume method
6. Power series
7. Spectral methods and etc...

The **variation method** and weighted **residual methods** generally employ differential equation for solving the problems.

**Finite element method** makes use of polynomial expression formulated with group of simultaneous equations in solving complicated problems of engineering structures

In the **field of FEM technique computers** are used to speed up the solution and solve very complex problem in a very short time.

### **Finite Element Method**

A complex problem (region of continuing or Domain) - is discretized in to simple problems (i.e. Into simple geometric shapes or subdomains ). These splitted domains are interconnected at some critical points.

Sub domains = finite elements

Inter connected points = nodal points or simply nodes

1. The material properties and
2. Governing relationships' (applied force and resultant displacement etc.)

These two things are imposed on the sub domains (finite elements) and suitable **simultaneous equations** are formed for all elements.

The solutions of these equations give the approximate behaviors of the continuum (domain)

**The sum of the elemental solutions will provide the required approximate solution for the whole domain or system.**

**FEM / FEA processes involve three stages of activity 1. PREPROCESSING 2. PROCESSING 3. POST PROCESSING.**

**preprocessing** - preparation of data (nodal co-ordinates, connectivity, boundary conditions, loading and material information)

**processing** – stiffness generation, stiffness modification and solution of equation, resulting in the evaluation of nodal variables.

derived quantities like gradients or stresses evaluated.

**post processing - presentation** of results.

deformed configuration, stress distributions, temperatures etc. are computed and displayed.

## **BASIC STEPS OF Finite Element Method**

- 1 Discretization of the structure**
- 2 Selection of Displacement function**
- 3 Formation of the element stiffness matrix and load vector**
- 4 Formation of Global stiffness matrix and load vector**



- 5 Incorporation of Boundary conditions**
- 6 Solution of Simultaneous equations**
- 7 Calculation of element strains and stresses**
- 8 Interpretation of the result obtained.**

**Discretization or Finite element zing and their type**

- 1. Based on Dimension**
- 2. Based on Material Property**
- 3. Based on Degree of Freedom**

#### **1 Based on Dimension**

**One dimensional element - line element , two dimensional element – Triangular and quadrilateral, three dimensional elements - Tetrahedral and hexahedral elements**

#### **2 Based on Material Property**

**Linear element , non linear element**

#### **3 Based on Degree of Freedom**

**Translational - one or two or three degree of freedom**

**Rotational - one or two or three degree of freedom.**

#### **Category of finite elements**

**Simple , complex and multiple elements**

**The order of polynomial used in interpolation function decided by the geometry of the element**

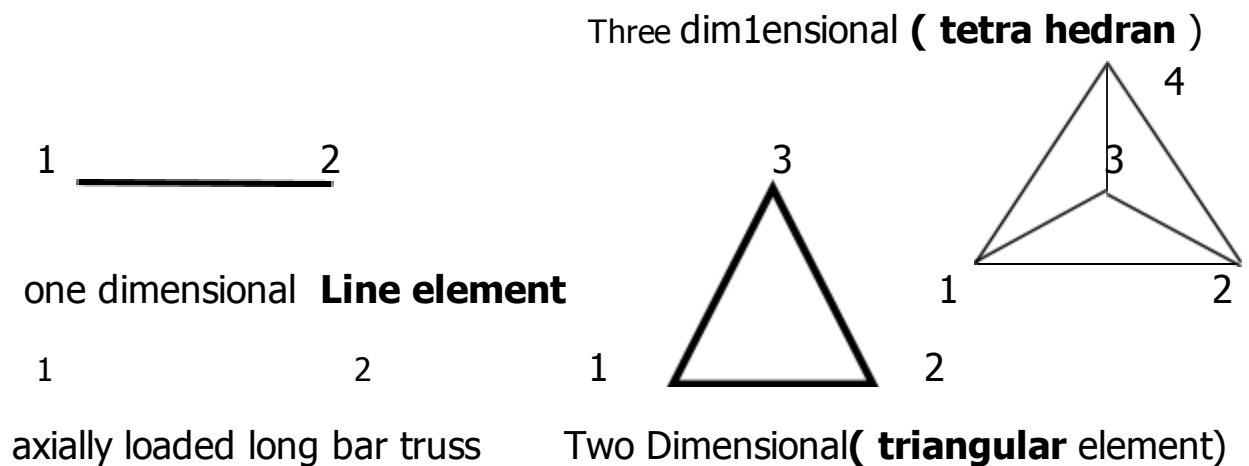
#### **For**

**Simple elements - Interpolation function contains only constant and linear terms only.**

Ie. **Approximating polynomial expression has only constant and linear terms in simplex element. So they are called linear elements.**

**The elements specified for  $n=1$  single order polynomial for one two and three dimensional elements.**

The simplex element of **one dimensional** is a line with two nodes at the ends



**Temperature distribution in rod and the pressure distribution in pipe flow are one dimensional problems.**

**The plates under biaxial loading**

### **Shape function**

In finite element method complicated structure is discretized into many elements and nodal displacements and stress in elements are combined in proper format to arrive overall displacement and overall stresses of structure inside the element. To find nodal displacement inside and in nodal points the mathematical expression namely Finite element equation or shape function were used. Using the values of nodal displacements, displacement at the interior points of element can be found out.

Displacement at the primary nodes evaluated using applied force with the nodal displacement in finite element equation.

The element **classifications** due to

Shape – **Geometry and the order of polynomial** used in interpolation are

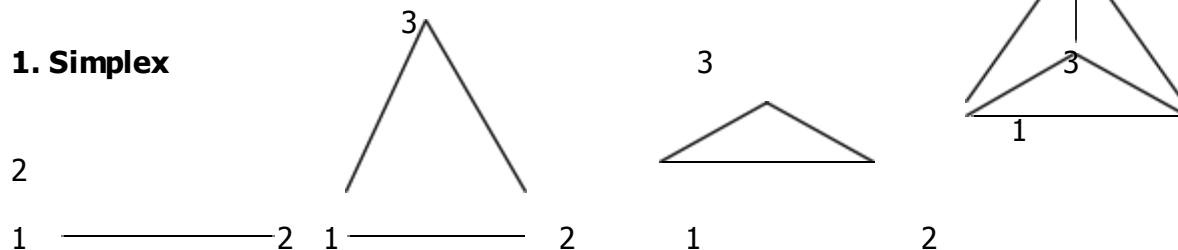
## 1. Simplex 2. Complex 3 Multiplex

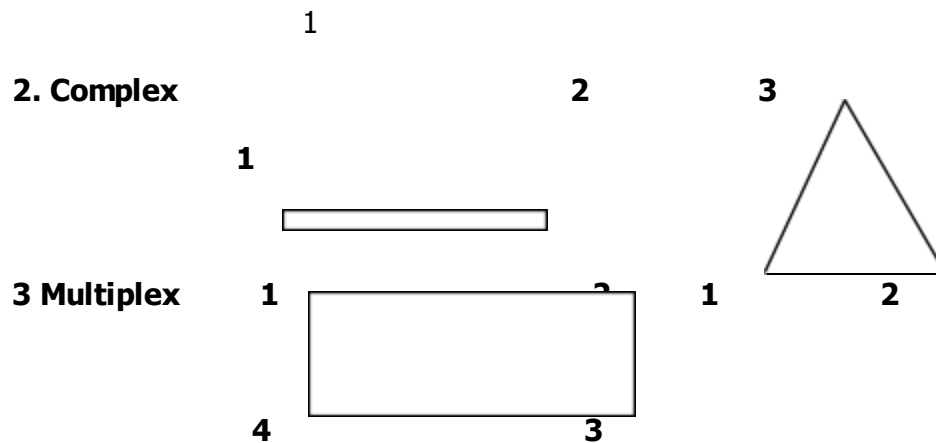
**Simplex** - Approximating polynomial expression (interpolation function) consist of **constant and linear** terms only. For **simplex element  $n=1$**  . example one dimensional straight line having 2 nodes 1 and 2 two dimension is flat triangle having 3 nodes 1 2 3 and the three dimension is tetrahedron with flat triangle surface with 4 corner nodes.

**Complex** – approximation polynomial expression consist of **quadratic , cubic and higher order terms** in addition to **linear and constant** terms.  **$N = 2, 3$**  or more for complex element. will have additional boundary node and sometime additional internal nodes. Other than primary nodes Number of nodes equal to number of polynomial coefficient (a1..am )

**Multiplex-** whose boundaries are parallel to the co ordinate axes to get inter element continuity. Approximation polynomial contain higher order terms

### 1. Simplex





## Stress and Equilibrium

Different types of stresses act on structural machine element when subjected to load

**1 Tensile stress 2 Compressive stress 3. Bending stress 4. Shear stress 5. Crushing stress**

These stress grouped into two major categories 1 Normal Stresses 2 Shear stresses

**These loads are due to the following forces**

|                    |        |                                |
|--------------------|--------|--------------------------------|
| Self weight        | —————→ | Body Force                     |
| Externally applied | —————→ | Point Load                     |
| External pressure  | —————→ | Surface Force = Traction force |

## For Analysis

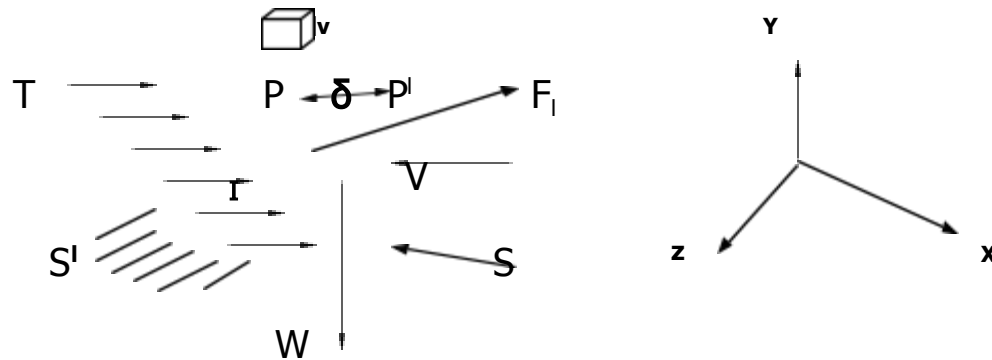
**Load and stress applied from many direction are resolved into three universal system of axes as X Y Z axes**

**When body or structure subjected to different type of forces is to be kept in stable condition = equilibrium condition or state**

**Therefore Algebraic sum of all the forces in X Y Z direction must be 0.**

$$\Sigma F_x = 0, \Sigma F_y = 0, F_z = 0$$

The same rule is applied for stresses also  $\Sigma \sigma_x = 0, \Sigma \sigma_y = 0, \Sigma \sigma_z = 0$ .



V- volume of body, S- surface, W- body force

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

**The points , forces, displacements etc. must be specified in terms of global axes of system X Y Z axes.**

**Therefore P specified as  $P_{(xyz)}$**

**F specified as  $F = (F_x, F_y, F_z)$**

distributed force per unit volume  **$f = (f_x, f_y, f_z)$**

**Surface force  $T = (T_x, T_y, T_x)$**

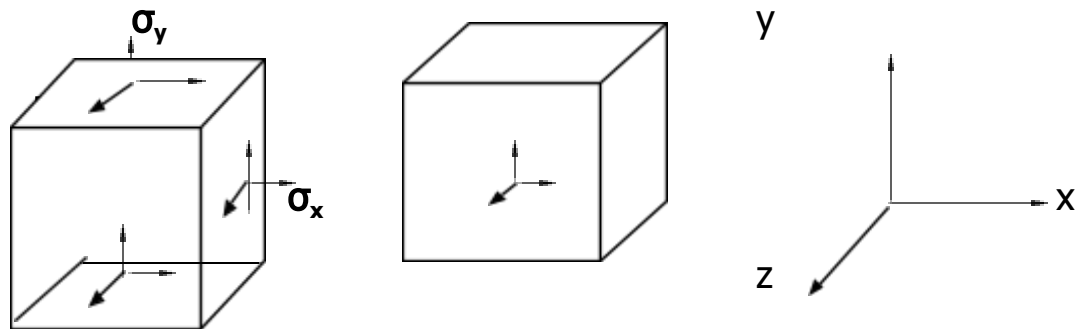
**Body force  $W = (W_x, W_y, W_z)$**

**Displacement  $\delta = (u, v, w)$  where  $u, v, w$  are displacement produced in  $x, y, z$  direction**

In elementary volume  $d_v$  induced stress represented by **independent component**

$$\text{Induced stress } \sigma = \sigma_x \quad \sigma_y \quad \sigma_z \quad \tau_{yz} \quad \tau_{zx} \quad \tau_{xy}$$

Where  $\sigma_x \quad \sigma_y \quad \sigma_z$  are **normal stresses** and  $\tau_{yz} \quad \tau_{zx} \quad \tau_{xy}$  are **shear stress**



The loading way of various locations and other surface conditions called **boundary conditions**

Suppose **Boundary conditions** can be specified as point load  $F$  of some magnitude applied Traction force acting at left and object fixed at left bottom end.

## Stress – Strain – Hooks law

Hooks law states displacement produced in elastic body is directly proportional to load applied up to its elastic limits.  $\sigma \propto e$   
Modulus of Elasticity ( $E$ )

$$\sigma_x \quad \sigma_y \quad \sigma_z \text{ Normal Stress } \tau_x \quad \tau_y \quad \tau_z \text{ Shear stress } e_x \quad e_y \quad e_z \text{ Normal strain } \gamma_{yz} \quad \gamma_{zx} \quad \gamma_{xy} \text{ Shear strain}$$

## Stress – Strain

$$e_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E}$$

$$e_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E}$$

$$e_z = \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}$$

$$Y_{yz} = T_{yz} \text{ and } Y_{zx} = T_{zx} \text{ and } Y_{xy} = T_{xy} \text{ and } G = \frac{E}{2(1+\mu)}$$

**E** is Modulus of Elasticity

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**G** is Modulus of Rigidity

**μ** is Poisson's ratio (ratio between lateral strain and linear strain)

For one dimensional objects if load applied in one direction along its axis

Stress related to strain is  $\sigma = Ee$

For two dimensional objects if load applied in two direction along its x and y axis

Plain Stress related to plain strain is

$$e_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} \quad e_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E} \quad e_z = \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E}$$

$$Y_{xy} = \frac{2(1+\mu)}{E} T_{xy}$$

Strain and Displacement

Displacement u v w produced in x y z direction BY applied stress

$e_x \ e_y \ e_z$  Normal Strain  $Y_{yz} \ Y_{zx} \ Y_{xy}$  shear strain

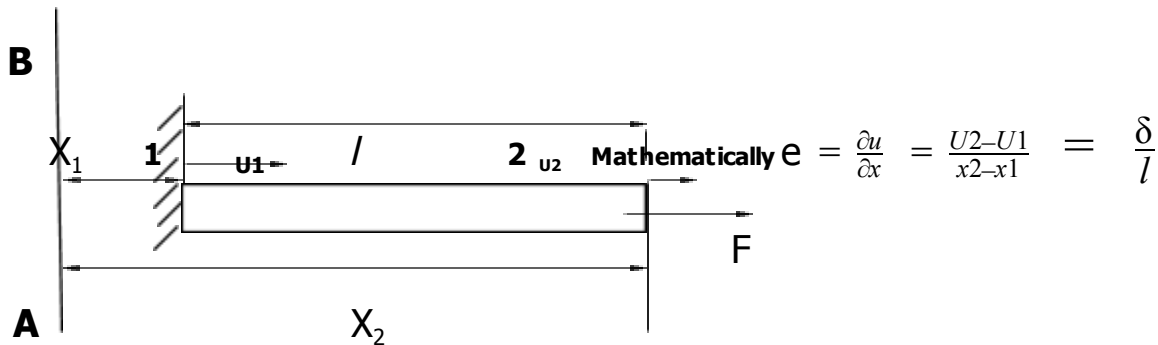
Normal Strain due to applied stresses are

$$e_x = \frac{\partial u}{\partial x}, \quad e_y = \frac{\partial v}{\partial y}, \quad e_z = \frac{\partial w}{\partial z}$$

Shear Strain due to applied stresses are

$$Y_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad Y_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \quad Y_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

**In an elastic body linear strain is change of displacement in specific length**



Where  $U_1$  displacement at end point 1 in x direction and  $U_2$  displacement at end point 2 in x direction from reference line A B

### **Elastic Constant**

Young's modulus E Shear modulus G Poisson's ratio  $\mu$  Bulk modulus K are considered as Elastic constant of an Elastic metal.

These elastic constants related with each other as  $E = 2G(1+\mu) = 3K(1+2\mu)$

Young's modulus is the ratio of normal stress to normal strain

Shear modulus is the ratio of shear stress to shear strain

**Poisson's ratio** is the ratio of lateral strain to linear strain

Bulk modulus is the ratio of applied stress to volumetric stress.